STRATEGIES USED BY YEAR 9 STUDENTS TO SOLVE PROBLEMS ON SPEED

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Since 1946 there have been a number of studies reported concerning students understanding of the concept of speed (e.g., Piaget 1946, Trowbridge 1979). This paper describes one part of a study concerning Year 9 students' strategies in solving problems which involved comparing the speed of two trolleys given the starting and finishing position and the time taken for each one. It was found that students used a variety of strategies ranging from guessing to intuitive use of variables to rather sophisticated uses of ratios and a calculation. Students also revealed some misconceptions concerning the variables that determine speed. Of particular interest was the use of time or distance as a determining variable without mention of the other variable.

Piaget (1970, 1946) studied young children's concepts of speed and motion using concrete illustrations. He was able to identify stages in the development of the concept of speed for "The gradual passage from intuitive thinking, still tied to the information of the senses, towards operational thinking, which forms the basis of reasoning itself" (p. ix). Trowbridge (1979) extended Piaget's ideas and questions in a study of first year University students. He was able to reveal "some of the ways in which college students in introductory physics courses think about velocity" (Trowbridge and McDermott 1980, p. 1028). The major misconception that these students had "was an inability to discriminate between position and velocity" (p. 1028).

This paper reports on a small component of a larger study involving secondary school students' concepts and responses to problems involving speed. The results are confined to the responses of students in an unstreamed Year 9 class to a set of questions called Dual Focus Closed Comparison Questions. Each question consisted of two diagrams representing the starting and finishing points of a trolley and the time taken for the trolley to get from its starting point to its finishing point. The students were asked to identify which trolley had the greatest speed and give an explanation of how they obtained their answer.

Originally ten questions were developed covering the ways that the variables: time, distance and starting points, could be arranged with the same and different speeds. In the final test, of which the Dual Focus Closed Comparison Question formed only a part, only five of these questions were used. These five questions are given in the appendix. Table 1 reports a summary of how the variables differ in each of the questions. Note that the two speeds in each question are the focus of the comparison.

Table 1
Variables in Questions

	Variables			
Question	**start position**	**time**	**distance**	**speed**
1	same	same	same	same
2	same	differ	differ	same
3	same	same	differ	differ
4	differ	differ	same	differ
5	differ	differ	differ	differ

At the time the test was administered 20 students were present: eleven girls and nine boys (average age 14 years, 6 months). The test was given during normal class periods. On completion of the preliminary analysis of the tests, interviews were conducted with five students. Both the test and the interviews were given without prior warning to the students. In the analysis that follows, the focus of attention is on the quality of each response as well as on the strategies of solving the problem implied by the responses.

RESULTS

Table 2 details a summary of the quality of responses that were given by the students tested. The responses were categorised into Full, Partial and Inappropriate explanations. A Full response occurred when a student referred to all variables that were required to obtain an answer. A partial response occurred when only some of the variables were referred to in the explanation. An inappropriate answer occurred when the response referred to some aspect of the question which did not reflect a correct approach to the problem. An answer that was correct but for which the explanation was inappropriate was considered to be an incorrect response.

Table 2
Type of Quality Response

Type of Response		Questions 1	2	3	4	5
correct responses						
- Full explanation	CF	12	5	9	7	7
- Partial explanation	CP	5	5	4	2	5
- no reason, or guess	CU	3	2	2	1	1
total correct		20	12	15	10	13
Correct response						
Inappropriate reason	CR	0	2	3	0	3
Incorrect responses						
- Full explanation	IF	0	0	0	1	1
- Partial explanation	IP	0	2	2	4	1
- Inappropriate reason	IR	0	4	0	3	1
- no reason or not done	IU	0	0	0	2	1
total incorrect		0	8	5	10	7

While a number of interesting features are revealed by this analysis, two warrant comment. First, Question 1 was included to test the students' ability to comprehend the diagram when all variables were identical. All students were able to do this. Second, more students were correct with Question 5 than Question 4. This was a surprise as in Question 5 all variables were different. The contingency table below shows the relationship of the correct responses to Questions 4 and 5.

		Q. 5 C	Q. 5 I	
Q.4	C	10	0	C = correct
	I	3	7	I = incorrect

From this table it can be seen that all students who had Question 4 correct also had Question 5 correct. The three students who had Question 4 incorrect and had Question 5 correct, gave only partial answers to both questions which in the case of Question 5 reflected a lack of understanding of the problem. During interview two students said that "(trolley) A got further" and one student said that "(trolley) B was behind by 1 sec".

STRATEGIES

A summary of the different strategies used by students to solve the questions is given in Table 3.

Table 3
Frequency of Strategies

Strategy	Frequency				
	1	2	3	4	5
1. Calculation	0	1	0	0	3
2. Ratio	0	4	1	3	1
3. Informal (time and distance)	11	6	10	6	4
4. Time only	2	4	1	3	1
5. Distance only	0	2	5	5	5
6. One in front	0	0	1	0	3
7. Faster/speed	3	1	0	0	0
8. Guess	1	1	1	0	1
9. Unknown	3	1	1	3	2

Below are some of the responses of the students using the different strategies in the speed test with the code for quality of response. Also included are some extracts taken from the interviews. (The Colon : separates the answer from the explanation)

1. **Calculation** Students actually calculated the speed of both trolleys using distance ÷ time.
   Student 920 Question 5 (CF)

   [Trolley] A: A $= \frac{5}{4} = 1.25$ secs units per sec. B $= \frac{3}{3}$ secs = 1 unit per sec so [trolley] A went faster.

It was interesting to note that the student did not quote a formula but merely divided distance by time to be able to compare the two ratios. In general it was only when the better students got to Question 5 that they reverted to a calculation approach. On analysing their previous responses it was found that all of these students had used the ratio approach at least once in other questions. They may not have needed to find the quotient of these ratios because at least one of the components of the ratio was the same. But with Question 5 all components of the ratio were different and one easy way to compare the speeds is to find the quotients of the ratios.

2. **Ratio** Students compared the ratio of distance and time for each trolley without evaluating the quotient.
   Student 906 Question 4 (CF)
   [Trolley] B: Because [trolley] A took 5 secs to go 3 units and it took [trolley] B 3 secs to go 3 units.
   Student 915 Question 2 (CF)
   Same [speed]: Because they are same speed by time and cm (equivalent).
   This student could give a response that was not tied to the actual values of the problem, i.e., he was able to focus on the variables themselves.
   Interview:
   S They are going the same speed at 1 sec a unit.
   I How did you get 1 sec a unit?
   S Um well in [trolley] A, 3 divided by 3 equals 1 that is 1 unit per sec and in [trolley] B, 5 divided by 5 gives 1, 1 unit per sec.
   Note that in the test and in the initial part of the interview this student used the ratio 'time : distance' but in the final part of the interview he reverted to 'distance : time" and calculated the quotient to find the speed, which indicates that, in this case, speed is neither a learned formula nor a triggered response.

3. **Informal** (Time and distance) Students used both time and distance in their response.
   Student 903 Question 3 (CF)
   [Trolley] B: it reaches the farthest [sic] in only 3 secs.

Interview:

S um··[trolley] B because it reached 5 in 3 secs and [trolley] A only reached 3 in 3 secs and 5 is going the fastest.
I Why? [can the student generalise and talk about the variables and not only the particular values?]
S Because it only took 3 secs to go to 5.
I But [trolley] A took 3 secs to get to 3.
S ··well I don't know um um the secs are shorter to get to a lower number.
I Are you saying it went a bigger distance?
S Yea in fewer secs.
I Righto? But the seconds are the same.
S Yea but ·· but in 3 secs [trolley] A only stopped at 3.
I Yes so [trolley] A stopped at 3.
S And [trolley] B took 3 secs and it got to 5.
I So [trolley] B had greater speed?
S Yes.

This student could not make the jump to using variables and kept on using the values associated with the variables.

Student 908 Question 4 (CF)

[Trolley] B: [trolley] A started at 2 [trolley] B started at 0 and [trolley] B got the same distance in less time.

This student started using the values of the variables in his explanation but was able to finish by using the variable names correctly.

Student 915 Question 5 (CF)

[Trolley] A greater: Quicker time for distance

This student is fairly typical of those who used this type of strategy for Question 5. It would appear that the strategy these students are using is: Pull trolley B back one unit so that it starts at the same place as A. It then reaches 3 units in 3 secs. So A should get to 4 units in 4 secs but it went to 5 units so it must be going faster, or it should get to 5 units in 5 secs but it only took 4 secs, i.e., "it had a quicker [shorter] time for [its longer] distance". This strategy of referring to both distance and time was the most common strategy that was used in the responses.

4. **Time only** Students used only time in their explanation.

Student 913 Question 2 (IR)

[Trolley] B: Because its taking 3 sec for [trolley]A.

This student has focused on the trolley that has the shortest time but has ignored the fact that Trolley B has also gone a larger distance.

Student 914 Question 2 (CP)

Same [speed]: Train A has only been on for 3 seconds but B has been on for five.

In contrast to student 913 this student has only referred to time in his response but he seems to be aware that the larger distance does make a difference. The students who referred to 'time only' in Question 2 were not consistent in using this strategy across questions. They also used 'distance only' and 'distance and time' strategies.

5. **Distance only** Students used only distance in their explanation.

Student 904 Question 2 (IP)

[Trolley] B: cause [trolley] B got further than [trolley] A.

Students who used this strategy seem to be focusing on the actual pictures that are given and seeing which trolley has gone the greater distance. It is similar to the next strategy where the student looks for the trolley that is in front.

6. **One in Front** Students used the term "in front" to explain the answer.

Student 917 Question 5 (CP)

[Trolley] A: [trolley] A is one second in front.

This answer uses a similar reasoning to that presented by student 915 above. Students who identify the object in front as having the greater speed are using visual information without processing the other information that is given.

7. **Faster** Students used the term "fast" or "faster".

Student 901 Question 2 (IR)

[Trolley] A: Because it is faster than [trolley] B.

The term "faster" might mean 'speed' but it is more likely to refer to 'time being shorter'. Other students often use 'speed' to refer to the object with the shortest time.

DISCUSSION

Students who were able to give a full explanation used the strategies of ratio, calculation and distance-time comparisons. In the interviews these students were generally consistent with the approach they took in the written test. Some students who gave a partial explanation in the test could give a full explanation in the interview with a little prompting. Most other students were found to be inconsistent with their approach to questions, trying to close quickly on a solution without giving proper regard to all the variables.

The most popular strategies involved the use of both the variables, distance and time. However at least six students focused only on one variable, either distance or time. They saw this as the determining variable and they were not consistent with the use of this variable across problems. They may have seen that one variable was held constant and then focused on the variable that was changing. An implication of this is the importance for the teacher to draw students' attention to all variables and discuss which ones are constant and which ones vary and how they interact with each other.

Student performance decreased with the increase of the number of variables that differ in the question. For those students who were more ratio-orientated, the final question "forced" them to do a calculation which was identical to the formula used to find speed. For the other students, the more the variables altered the more they had difficulty in identifying the speed and explaining it in a clear way.

One other aspect of the students' responses was the observed confusion over the meaning of the terms 'speed', 'fast(er)', 'quick(er)', 'shorter', 'time'. Often these words were used synonymously. In interpreting their responses this confounding feature had to be taken into account.

CONCLUSION

It is interesting to note that most students attempted these problems in an intuitive way. There was not a standard learned method of attack and most did not use mathematics in an algorithmic sense. Perhaps the style of presentation of the questions contributed to this. The results showed that students in Year 9 are not limited to one method of attack. Even individual students exhibit a range of strategies across questions. There was even marked differences in the variables that students chose to focus on in an attempt to solve problems. Students who could respond by referring to the variables and how they were related gave a higher quality response than those who attempted to explain by using values only. Some students used a mixture of the two which suggests a transition phase from 'value to variable thinking'.

Finally, the style and content of the questions proved to be most valuable in revealing substantial variability in student strategies. Further research is currently underway to capitalise on this work. In particular, the focus is on identifying the triggers that elicit different strategies and, more importantly, what features encourage an intuitive rather than algorithmic response.

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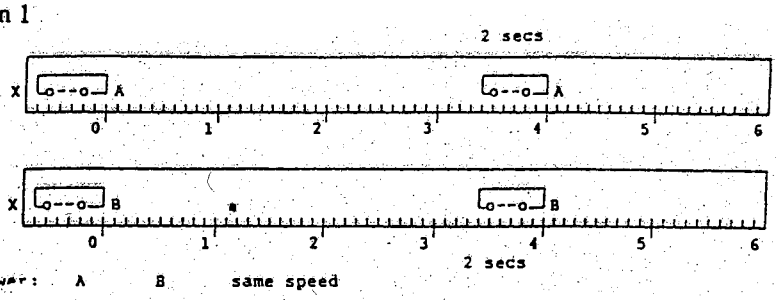
Appendix - Five Questions

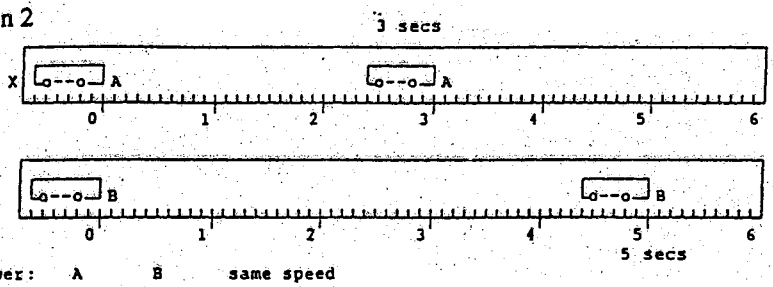
You must decide EITHER which cart is going the GREATER SPEED
OR if they are going the SAME SPEED.
Then explain your choice of answer.

Question 1



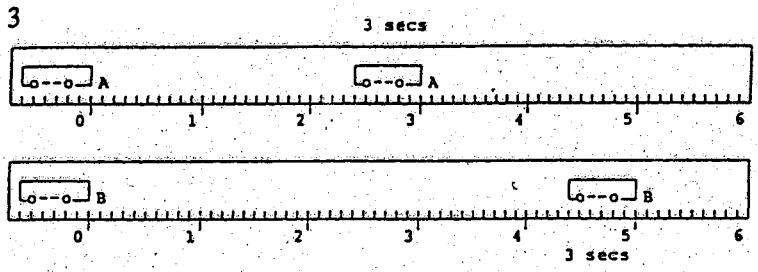
Answer: A B same speed

Question 2



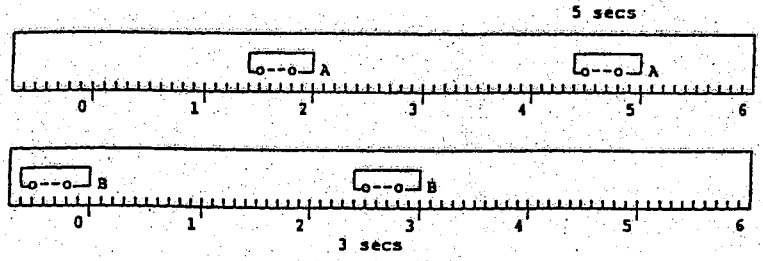
Answer: A B same speed

Question 3



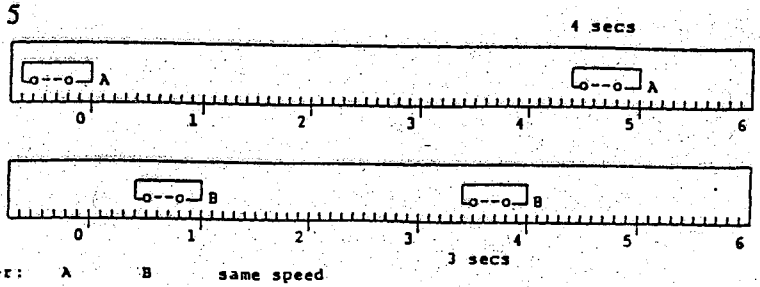
Answer: A B same speed

Question 4



Answer: A B same speed

Question 5



Answer: A B same speed